

THE ONSET OF TENSILE INSTABILITY

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Key Words:

Tensile test, mechanical properties, Young's modulus, yielding, strain hardening, geometric softening, necking, modulus of toughness, modulus of resilience, fracture.

Prerequisite Knowledge:

Basic knowledge of mechanical properties in tension, digital data acquisition and spreadsheets.

Objective:

Introduction of mechanical properties obtained from tensile testing with an emphasis on being able to fully analyze the data, in particular the phenomenon associated with the onset of necking.

Equipment:

1. Cylindrical tensile specimens
2. Extensometer
3. Tensile test system of suitable load capacity and which acquires data digitally.
4. A computer with Microsoft Excel, Corel Quattro Pro, or another suitable spreadsheet program.

Introduction:

In this experiment the plastic deformation behavior and the onset of plastic instability of some common structural alloys is investigated. The usual mechanical properties are measured from stress-strain curves, but in addition, strain hardening behavior and the onset of plastic instability are studied. Much more is happening during a tensile test than is listed in standard materials properties data tables. The goal of this experiment is to investigate these additional phenomena.

Elastic Deformation: When an isotropic material is loaded to stresses below the yield point and then unloaded again all of the deformation is immediately recovered. This behavior is usually associated with the linear portion of the stress-strain curve, although not all materials exhibit this linear-elastic behavior; for example, rubber is an exception. But for many materials the resistance to deformation in this region is given by Young's modulus Y , which also is often represented by an E , is simply the slope of the stress-strain curve in the linear region

$$Y = \frac{\Delta\sigma}{\Delta\epsilon}. \quad (1)$$

where F is the true stress and ϵ is the true strain. When measuring Young's modulus one must take into account the stiffness of the testing machine. Ideally, the machine should be perfectly

Young's modulus above is given here in terms of the slope of the true stress-strain curve. In many cases it is measured from the slope of the load-elongation () F -) L)

$$Y = \frac{\Delta F}{\Delta L} \frac{L_0}{A_0} = \frac{\Delta S}{\Delta e}$$

or engineering stress-strain () S -) e) curve. (A_0 is the initial cross-sectional area of the specimen, L_0 is the initial gage length.) ASTM standard E 111-82 recommends using this method unless the strain exceeds 0.25% in which case this standard recommends taking into account the changes in gage length and area, in other words, the true stress-strain curve. The differences in the two values of Young's modulus is only about 0.5% for steel ($Y=209$ GPa).

stiff but in reality its stiffness may be in the neighborhood of 200,000 pounds per inch. A low machine stiffness will introduce errors in the measured modulus as given by

$$Y_{measured} = Y \left[1 + \frac{A_0 Y}{KL_0} \right]^{-1} \quad (2)$$

where A_0 is the initial cross-sectional area of the specimen, L_0 is the initial gage length, Y is the correct value of Young's modulus and K is the stiffness of the testing machine [1]. The error in the measured modulus results primarily from errors in measurement of elongation. The specimen is a monolithic part while the testing machine has the grips, clevis pins, threaded parts and moving parts that mesh better when loaded means the specimen is generally stiffer than the testing machine. Consider also that the modulus is the ratio of stress, a large number, to strain, a small number. The result is very sensitive to errors in the denominator.

When conducting a constant crosshead speed test one can compute the stiffness of the testing machine K using the equation

$$K = \left(\frac{\dot{x}}{\dot{F}} - \frac{L_0}{A_0 Y} \right)^{-1} \quad (3)$$

where \dot{x} is the crosshead speed and \dot{F} is the loading rate [1]. Or, one can use an extensometer to measure strain directly from the specimen, or one can attach strain gages to the specimen.

Yielding: Yielding occurs when deformation changes from being mostly elastic to mostly plastic. By this definition the method of measuring the yield strength may seem somewhat arbitrary, which it is, however, standard methods have been established so that everyone will get the same result. In fact, there are a number of standard methods available and the method used varies depending on the type of yielding behavior (gradual, sharp, upper/lower), industry, or country. Offset techniques are used when yielding is gradual while sharp and upper/lower type yielding behaviors are easily read from the stress-strain curve. In the case of the upper/lower yielding behavior which is characteristic of carbon steels and a few other alloys the lower yield stress is

taken as the yield strength due to the fact that (1) accurate measurements of the upper yield strength are difficult to make and (2) the lower yield stress is the stress where normal, homogeneous plastic deformation begins.

Strain Hardening: Plastic deformation can occur by three different processes: diffusional processes, twinning and dislocation motion. The contribution of diffusional processes is important at high homologous temperatures, $T > 0.6 T_{mp}$, and twinning might account for up to a few per cent strain. Dislocation processes are the dominant deformation mechanism of plastic deformation at normal temperatures. Considering only the contribution due to dislocation processes, a shear stress acting parallel to the slip plane causes these dislocations to move through the lattice. As they move they may encounter obstacles such as solute atoms, particles, grain boundaries and other dislocations, effectively impeding their motions until the stress is increased enough to allow the dislocation to overcome the obstacle. As the dislocations glide, climb and cross-slip through the lattice additional dislocations were being generated and start moving through the lattice. With this ever increasing dislocation density the distance between them decreases, they encounter each other more often, and it becomes increasingly difficult to deform the material. This process is called strain hardening.

There are two well known equations that depict the strain hardening phenomenon. The Hollomon equation describes strain hardening as a power law function of stress and strain after yielding. Hollomon's equation is

$$\sigma = K\epsilon^n \quad (4)$$

where F and ϵ are true stress and strain, n is the strain hardening index and K is the strength coefficient which is equal to the stress at $\epsilon = 1$. The second equation is Ludwik's equation which is often preferred because, unlike the Hollomon equation, it does not suggest that strain hardening begins at the very start of the tensile test. It includes a term for the yield stress F_0

$$\sigma = \sigma_0 + K\epsilon^n \quad (5)$$

where F_0 is given by

$$\sigma_0 = \left(\frac{K}{E^n} \right)^{\frac{1}{1-n}} \quad (6)$$

In cases where the material has already experienced some plastic deformation (ϵ_0 =prior strain) the following equations represent strain hardening:

$$\sigma = K(\epsilon + \epsilon_0)^n \quad (7)$$

$$\sigma = \sigma_0 + K(\epsilon + \epsilon_0)^n \quad (8)$$

The strain hardening index n is a constant for a given material and can range in value from 0 to 1 but is typically in the range of 0.2 to 0.5. The strain hardening index can be written as

$$n = \frac{d \log(\sigma)}{d \log(\epsilon)} = \frac{\epsilon}{\sigma} \frac{d\sigma}{d\epsilon} \quad (9)$$

and it can be evaluated numerically or graphically from the slope of a plot of $\log(\sigma)$ vs- $\log(\epsilon)$ where n is equal to the slope of the resulting line. If the Hollomon equation is obeyed the line will be straight, but if the Ludwik equation represents the data or the specimen had been deformed prior to the tensile test the line will be concave up (figure 1). In these cases the values of F_0 and ϵ_0 must be known in order to determine the value of n .

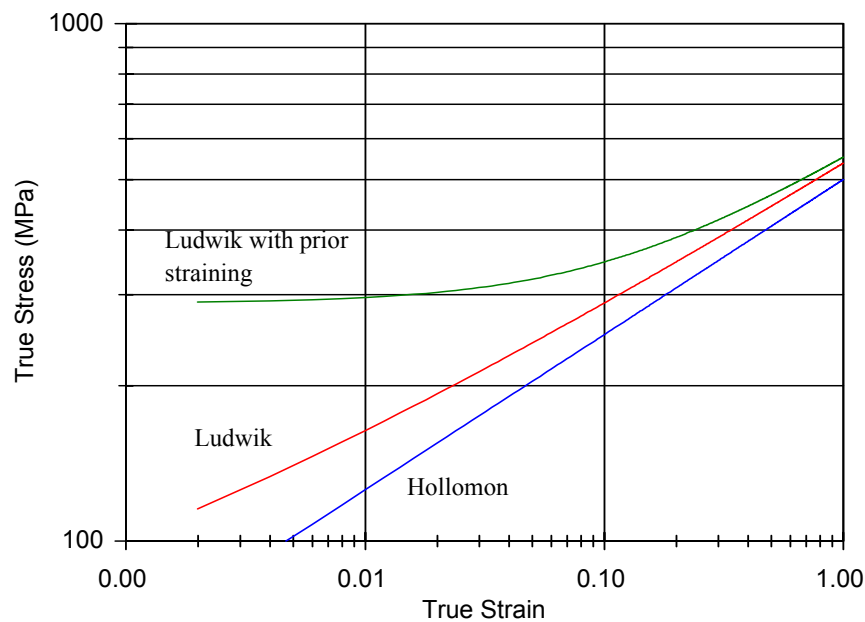


Figure 1. Log-log plots of stress-strain curves calculated using Hollomon's equation, Ludwik's equation and Ludwik's equation with 10% prior straining. ($K=500$ MPa, $n=0.3$, $E=207$ MPa)

The above equation allows one to express strain hardening $d\sigma/d\epsilon$ in terms of the rate of increase of stress with respect to strain. Rewriting it as

$$\frac{d\sigma}{d\epsilon} = n \frac{\sigma}{\epsilon} \quad (10)$$

shows that the rate of strain hardening (slope of the stress-strain curve) is proportional to the stress divided by strain.

Strain Rate Hardening: Most materials are sensitive to the rate of deformation, especially at

elevated temperatures. This behavior often obeys a power law expression such as

$$\sigma = A\dot{\epsilon}^m \quad (11)$$

where A is a constant at given temperature and metallurgical condition, $\dot{\epsilon}$ is the true strain rate and m is the strain rate sensitivity. This equation can be combined with the Hollomon equation for a general description of the plastic deformation of materials

$$\sigma = B\dot{\epsilon}^m \epsilon^n. \quad (12)$$

At low and ambient temperatures the rate of strain hardening is significant, $0.2 < n < 0.5$, while the rate sensitivity is small, $0.001 < m < 0.01$. At higher temperatures just the opposite is true and the strain rate sensitivity can reach values as high as 1.0 (Newtonian viscosity) while the strain hardening index may approach zero. A value of $m=0.5$ is considered characteristic of superplasticity, a condition where elongations of 100's even 1,000's percent strain are possible due to the stability of the neck that may form at the UTS.

While the strain rate sensitivity can effect the flow stress, it is more important in terms of its effect on ductility. Increased strain rate sensitivity slightly increases the strain at which the material starts to neck, but more significantly it can extend ductility considerably by stabilizing the neck. As the specimen begins to neck the strain rate in the necked region increases and due to the strain rate sensitivity the stress in this region increases and thus the neck stabilizes. So while the strain hardening index is a measure of a material's resistance to the onset of instability, the strain rate sensitivity is a measure of its resistance to necking.

Tensile Strength: The tensile strength S_{TS} [formerly called the ultimate tensile strength (UTS) but now considered an obsolete term] is the maximum nominal stress. This property represents the load bearing capacity of the material. It also occurs at the strain where necking begins. The subsequent drop in stress beyond this point is due to the *geometric softening* associated with the localized thinning and deformation of the specimen.

If a true stress-true strain curve could be obtained directly from a tensile test, rather than an engineering stress-strain curve, one would find that there is no maximum stress prior to failure. There would appear to be no tensile strength. That is not to say that necking has not occurred or that there is no tensile strength, but that other methods must be employed to determine its value and the strain at which it occurred.

There are several methods for determining the tensile strength F_{TS} from a true stress-strain curve. One method is to plot both the true stress and the rate of strain hardening against true strain as shown in figure 2. The point where these two lines intersect marks the tensile strength and the strain where necking begins. This method is expressed in the following equation

$$\sigma = \sigma_{TS} \text{ when } \frac{d\sigma}{d\epsilon} = \sigma. \quad (12)$$

While this relationship may be derived from the Hollomon equation it also works for stress-

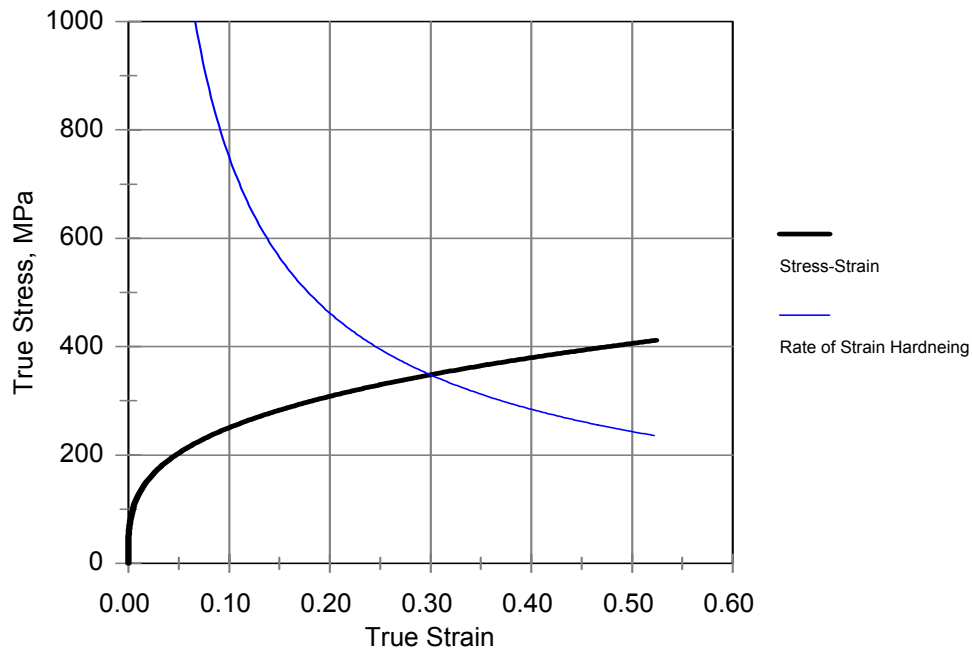


Figure 2. Stress-strain and rate of strain hardening curves calculated using the Hollomon equation. (K=500 MPa, n=0.3)

strain behaviors which do not obey the Hollomon equation.

The above equation also gives us a second method of determining the tensile strength – the unit subtangent method. Using this method the tensile strength may be found by locating a point on the true stress-true strain curve which has a subtangent of unity (intersects the strain axis at strain $\epsilon_{TS} - 1$). Furthermore, it may be shown that

$$\sigma = \sigma_{TS} \text{ when } \frac{d\sigma}{d\epsilon} = \frac{\sigma}{1+\epsilon}. \quad (12)$$

This is Considère's criterion, a third method for determining the tensile strength. It is also a graphical method that requires a plot of true stress versus nominal strain whose origin is set at the yield point. The tensile strength F_{TS} is obtained by drawing a subtangent from a nominal strain of -1. The nominal tensile strength S_{TS} is the point where this line intersects the stress axis. (See figure 3.) If power law hardening is observed then Considère's criterion may also be used to derive one last expression for the tensile strength

$$S_{TS} = K n^n (1-n)^{1-n}. \quad (13)$$

Onset of Plastic Instability: Generally, the strain at the maximum load marks the onset of plastic instability. For annealed materials the rate of strain hardening immediately after yielding is

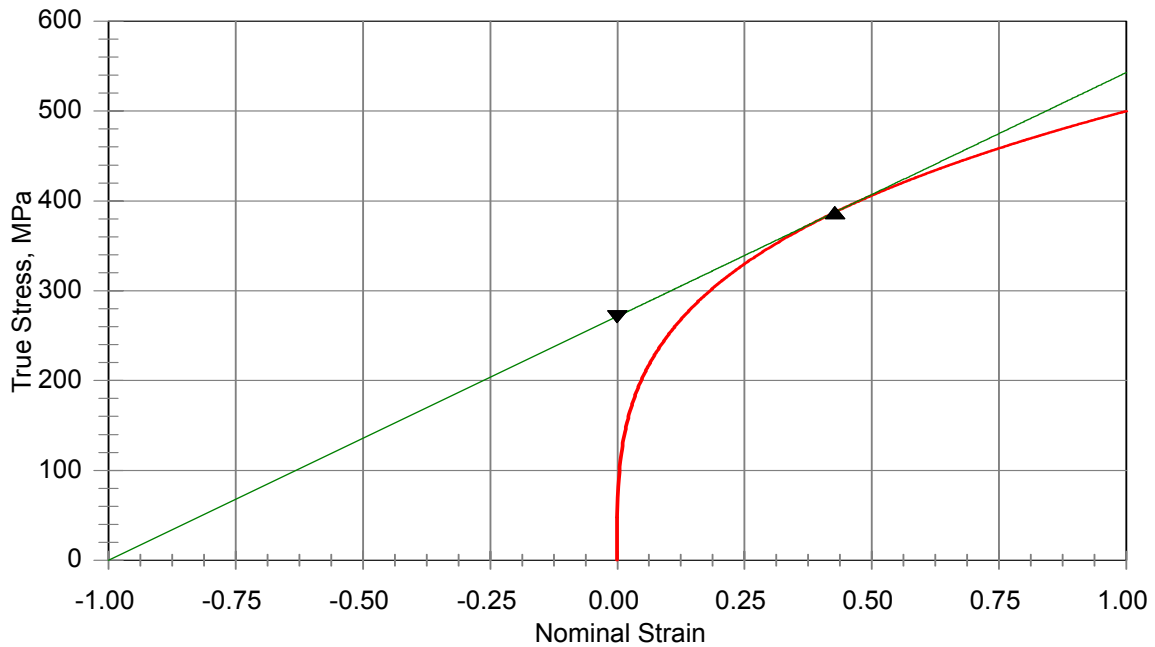


Figure 3. Considère's construction for determining the tensile strengths ($S_{TS} = 271$ MPa, $F_{TS} = 388$ MPa) of a ductile material. The stress-strain curve was calculated using Hollomon's equation ($K=500$ MPa, $n=0.3$).

relatively high, but as deformation continues, the rate of strain hardening steadily decreases. At the same time the specimen is becoming thinner. Eventually the rate of strain hardening and specimen thinning are equal. Beyond this point the rate of specimen thinning is greater than the rate of strain hardening and deformation is unstable, i.e., necking begins. The instability will start in some part of the specimen where an inhomogeneity (soft spot, adjacent to a hard spot) or at a machining defect (stress concentration) exists. Once localized deformation begins the stress in this region will be higher than elsewhere, causing deformation to become even more localized, leading to the formation of a macroscopic neck.

For rate insensitive materials, the onset of unstable deformation occurs at the strain where maximum load is obtained. It was mentioned earlier that the strain hardening index is a measure of the resistance to the onset of instability. It can be shown that the strain hardening index for rate insensitive materials is equal to the true strain where necking begins.

$$\epsilon_{TS} = n \quad (14)$$

For rate sensitive materials the onset of necking will differ but only slightly. The most important aspect of rate sensitivity is that it tends to stabilize the neck, decreasing its degree of localization, and extending ductility well past ϵ_{TS} .

Ductility and Localized Deformation: During nonuniform deformation stresses are concentrated

in the necked region of the specimen, leading to a more complex, triaxial, stress state. Bridgeman [2] has developed a method for computing the stress distribution in the necked region and making corrections to the stress-strain data based on the radius and area at the root of the neck.

The onset of instability often marks the limit of useful deformation. However, neck growth is slow at first and additional straining can occur before the instability is either noticeable or significant. The amount of straining beyond this point is affected primarily by the strain rate sensitivity of the material. For example, maximum load is often reached at about 10-15% strain, which is usually around half the strain to fracture for rate insensitive materials ($m = 0.01$). Superplastic materials, however, have high rate sensitivities ($m = 0.5$) and may be strained hundreds and even thousands of percent. The maximum load, however, occurred at around 10% strain. The high rate sensitivity stabilized the neck.

Fracture: With continued plastic deformation a material accumulates damage associated with dislocation processes. Dislocations that pile up at grain boundaries and other obstacles can form microcracks. Tensile stresses can open these into voids which grow and eventually coalesce, forming larger cavities. Final fracture occurs when the number and sizes of internal voids is sufficient to exceed the material's ability to withstand the applied load. These voids introduce stress concentrations which can lead to the spontaneous growth of a single crack. (This behavior is the subject of the field of fracture mechanics.) Inspection of the fracture surfaces permits determination of the fracture process involved, i.e., brittle, shear, or ductile. Close inspection of the cup-and-cone fracture surfaces of a ductile material may reveal areas where void formation and coalescence occurred and the area through which the final rupture occurred. A scanning electron microscope is usually required to determine whether the crack propagated intergranularly or transgranularly and how much plastic deformation occurred as the crack propagated.

The strain to fracture is not an intrinsic property of the material. It also depends on the specimen geometry. An expression that relates specimen geometry to ductility is

$$\epsilon_f = C \frac{A_0^{1/2}}{L_0} \quad (15)$$

where C is a proportionality constant [3]. Ideally, comparisons of strain to fracture should be made using specimens of identical design. If this isn't possible, the comparisons should be made on similarly shaped specimens. Corrections based on the $A^{1/2}/L$ ratio may be made. ASTM 370 describes how to make comparisons of round and flat specimens.

Procedure:

Preparation

1. Plot true stress versus true strain for a hypothetical material using either the Hollomon or Ludwik equation. Experiment with the values of K and n to see how their values effect the results. Note the value of the stress as true strain approaches 100%.

2. Determine the tensile strength for the stress-strain curve in the question above.
3. Calculate and plot an engineering stress-strain curve using the data calculated in question 1.
4. Compile mechanical properties data on the specific materials to be tested.
5. Note the specifications of the tensile tester and compare them to the load and elongation requirements for the tensile tests.
6. For the crosshead speed you plan to use, determine the data acquisition rate you will need in order to record enough data in the elastic region to be able to determine Young's modulus.

Materials

Specimens made of plain carbon and low alloy steels, aluminum, and plastics are available. All specimens are round with threaded ends. They all have a 2 inch gage lengths and either an 0.236 or 0.505 inch diameter.

Tensile Testing

Inspect the equipment and become familiar with how it operates. Check the calibration of the load cell and extensometer and make adjustments as required.

Select a specimen to test and measure and record its gage length, diameter, etc. Note the specimen's composition and processing, and then estimate the maximum load that will be generated during the tensile test.

Tensile test the specimen to failure. During the test monitor the load and elongation and watch the specimen carefully to see when necking begins.

After the specimen fractures, remove the broken halves and note the geometry of the fracture and the fracture surfaces. Do not try to fit the halves of the specimen back together again as this will damage the fracture surfaces. You'll probably want to examine the fracture surfaces under a low power microscope or a scanning electron microscope.

Data Analysis

Perform a comprehensive analysis of the data from the tensile tests by importing the data into your spreadsheet program. Construct the stress-strain curves and measure all of the usual mechanical properties including the energy capacities of the material. Analyze the strain hardening behavior and finally, investigate the onset of tensile instability.

Stiffness: Determine the value of Young's modulus using a regression or best fit routine. How does your value compare to the published value?

Optional: Repeat this analysis using data from a tensile test which did not utilize an extensometer. Compare this value to the previous value. This value is typically a factor of 10 to 20 lower than the value obtained using an extensometer. Determine the stiffness of the testing machine.

General Mechanical Properties: Compute the other basic mechanical properties that may be obtained from a tensile test.

Stress

Yield strength, (upper and lower, offset, etc.)

Tensile strength (stress at maximum load)

Stress at fracture

Strain

Strain at yielding

Strain at the onset of necking (observed and from the stress-strain curve)

Strain at failure

Reduction of area at failure (measured from the specimen)

Energy capacity

Work required to cause failure (area under the whole load-elongation curve)

Modulus of resilience (area under the elastic portion of the stress-strain curve)

Modulus of toughness (area under the whole stress-strain curve)

Compare these properties to those in reference books.

Strain Hardening: Use a least squares or regression analysis to determine the values of n and K in the Hollomon equation. Comment on the values obtained. Are these typical values? Does the Hollomon equation do a satisfactory job of representing the stress-strain behavior of these materials?

Tensile Strength: Determine the tensile strength using the engineering stress-strain data. Next, determine the tensile strength using the rate of hardening method. Finally, locate the tensile strength on the true stress-strain curve using only the value of the strain hardening exponent. Do all three methods give the same answer?

Optional: Construct a graph similar to figure 3 to determine the tensile strength.

Onset of Instability: At what point in the test did you first observe necking of the specimen? Compare this to the strain where maximum load (tensile strength) occurs on the stress-strain curve. Do the Hollomon strain hardening index and strain at the onset of instability coincide?

Fracture: Examine the fracture surfaces of each specimen. Did fracture occur in a ductile or brittle manner?

Measure the final cross-sectional area of the fractured specimen and compute the true stress at fracture. Compare this to the engineering stress that was obtained at the same strain and the true stress that the Hollomon equation would predict. One would expect it to be lower. Why? Was it?

Comments:

The analysis of the data requires mostly basic spreadsheet skills and the ability to lay out the solution to problem in a sensible and organized manner. A typical spreadsheet will include a header section (owner, file name, creation and revision dates, etc.), a parameters section (gage length, gage area, crosshead speed, etc.), and a results section where the results of the analysis of the data are listed. The bulk of the data will be in 10 or more columns containing, starting with column 1, the original load-elongation data, elongation data corrected for the toe of the curve, load-elongation data converted to the desired units, nominal stress-strain calculated from the load-elongation data, true stress-strain data calculated from the nominal stress-strain data, and additional columns which are required to measure selected mechanical properties. Building this spreadsheet can be quite a task unless one lays it out in a simple and direct manner.

The difficulty of the analysis of the mechanical properties from the columns of raw data can range from the use of simple @max() and @sum() functions to the more advanced regression analysis procedures. Other clever techniques may have to be devised to be able to determine the offset yield strength (if appropriate). In addition, one must also construct appropriate graphs that both illustrate each property and verify that the analysis has been done correctly. For example, after using a regression routine to determine the value of Young's modulus, one should then calculate the elastic portion of the stress-strain curve using this value and plot it along with the experimental data. One should then calculate the linear elastic portion of the stress-strain curve again but this time applying a strain offset so one can see where this line crosses the experimental stress-strain curve. To accurately locate the offset yield stress one can then subtract the measured stress from the calculated stress and then search the column of data to find the value closest to zero. Similar procedures and graphs should be used to determine the tensile strength using the rate of hardening method, for fitting the Hollomon equation to the strain hardening region of the stress-strain curve, etc.

Employing the rate of strain hardening method to determine the true tensile strength and the onset of instability requires plotting the slope of the true stress-strain curve. Due to the noise in real experimental data the result will be unacceptable unless a smoothing technique such as box car smoothing is employed. We have found that averaging over 21 and sometimes many more data points gives good results.

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Bibliography:

Michael L. Meier received his B.S. in Materials Engineering from North Carolina State University in 1979 and his M.S. (1986) and Ph.D. (1991) in Materials Science and Engineering from the University of California, Davis. After a two-year post-doctorate position at the Universität Erlangen-Nürnberg in Erlangen, Germany he returned to UC Davis to set up new research laboratories and to manage the laboratory teaching program. He is now the director of Materials Science Central Facilities.

Amiya K. Mukherjee received his M.S. degree in Physical Metallurgy from the University of Sheffield (1959) and D.Phil. degree in Metallurgy and Science of Materials from Oxford University, England (1962). From 1962-65 he was associated with the Lawrence Berkeley National Laboratory and the Department of Materials Science at the University of California, Berkeley, where he worked on application of dislocation theory to the understanding of thermally activated deformation mechanisms in crystalline materials. He was a Senior Scientist at the Battelle Memorial Institute in Columbus, OH (1965-66) primarily working on fracture and failure processes in engineering materials. In 1966 he was the first faculty recruited in the area of materials science at the University of California, Davis and he has been instrumental in developing the materials science program with his colleagues there since. He was promoted to full professorship in 1970.

His current research activities are in the areas of creep properties of metal and intermetallics, superplasticity, high temperature cavitation and failure phenomenon, synthesis, characterizations and processing of nanometals for enhanced plasticity including superplasticity, elevated temperature deformation characteristics of nanocrystalline thin film metallic multilayers, and processing and properties of nanoceramic composites for improved fracture toughness, creep resistance as well as superplasticity. Mukherjee has published over 350 research papers in journals and conference proceedings. He is a past Chairman of Materials Science Division of ASM International and of the Editorial Board of Metallurgical Transactions. He received the Distinguished Teaching Academic Senate Award, University of California, Davis (1978); Special Creativity Award for Research, NSF (1981); Alexander von Humboldt Senior Scientist Award, Germany (1988); AT&T Foundation Award from Amer. Soc. for Engineering Education (1988); University of Tokyo Centennial Gold Medal (1990); Albert Easton White Distinguished Teaching Award, ASM (1992); University of California Prize and Citation for Distinguished Teaching and Scholarly Achievement (1993); Pfeil Medal and Prize for Creativity in Research, Institute of Materials, U.K. (1994); the Bochvar Medal in Metals Physics from the University of Moscow (1996), Humboldt Professor, Max Planck Institute, Germany (1997); and Senior Fellow, Ecole Polytechnique Federale Lausanne, Switzerland (2000).

Appendix A - Symbols, Conversion Factors and Equations for Stress and Strain

Symbols

Symbol	Description	Preferred Units
F	Force	N
L	Length	mm, cm or m
D	Diameter	mm, cm or m
A	Area	mm ² , cm ² or m ²
S	Engineering stress	MPa
e	Engineering strain	-
F	True stress	MPa
,	True strain	-
t	Time	s

Conversion Factors

Description	Conversion Factors
Mass	1 kg = 2.207 lbs
Force	1 N = 10 ⁶ dynes = 0.2248 lbf
Length	1 in = 25.4 mm = 2.54 cm = 2.54x10 ⁻² m
Area	1 in ² = 645.16 mm ² = 6.4516 cm ² = 6.4516x10 ⁻⁴ m ²
Stress, pressure	1 MPa = 1 MN/m ² = 145 psi
Energy, work	1 J = 10 ⁷ ergs = 6.242x10 ¹¹ eV = 0.239 cal

Stress and Strain

	Nominal	True
Stress	$s = \frac{F}{A_0}$	$\sigma = \frac{F}{A}$
Strain	$e = \frac{\Delta L}{L_0}$	$\epsilon = \int_{L_0}^{L_f} \frac{dL}{L} = \ln\left(\frac{L_f}{L_0}\right)$
	$e = -\frac{\Delta A}{A_0}$	$\epsilon = \ln\left(\frac{A_0}{A_f}\right) = 2\ln\left(\frac{D_0}{D_f}\right)$ (See note 1)
Strain Rate	$\dot{e} = \frac{\Delta e}{\Delta t} = \frac{1}{L_0} \frac{\Delta L}{\Delta t}$	$\dot{\epsilon} = \frac{d\epsilon}{dt}$

Note 1: Constant volume is assumed ($A_0L_0 = A_fL_f$).

The true stress and strain can be written in terms of the nominal stress and strain, as long as the specimen has not started necking, by the equations

$$\sigma = s(e+1)$$

and

$$\epsilon = \ln(e+1).$$

Elastic Moduli

Modulus	Equation
Young's Modulus	$E, Y = \frac{\Delta\sigma}{\Delta\epsilon}$
Poisson's Ratio	$\nu = -\frac{\epsilon_x}{\epsilon_z}$, For constant volume $\nu = 0.5$
Shear Modulus	$G = \frac{\Delta\tau}{\Delta\gamma} = Y \frac{1}{2(1+\nu)} = K \frac{3(1-2\nu)}{2(1+\nu)}$
Bulk Modulus	$K = \frac{\Delta\sigma_{hyd}}{\frac{\Delta V}{V}} = G \frac{2(1+\nu)}{3(1-2\nu)}$

Appendix B - Sample Data from the Instron 4204 Universal Tester, Header and First 10 Data Points

```

Sample id : ALUMINUM           Test date : 30 Dec 1994
Version : 1.08                Version date : 30 May 1991
Machine : 4200                Robot used : NO
Report file# : 45            Operator : Mike Meier

X conversion : .03937008      X A/D offset : .0000
Y conversion : *****      Y A/D offset : .0000

Sample rate : 5.00           Extensometer : STD
2nd Sample rate : 1.00      Autostart : OFF
A/D range : 0               Geometry : RECTANGULAR
Calib type : AUTOMATIC      Calib extens : .0000
Calib load : 11240.4500     Humidity : 50
Temperature : 25           Units type : SI
Xhead speed : .1000        # specimens : 1
Test type : TENSILE        Entry dimens : YES
Bar type : E-45            Thresh delay : 4.49618
Break check : 10.00000     Extens limit : .98425
Load limit : 11240.45000

```

```

Sample dimensions :
A: .2500 B: .0559 C: 1.0000 D: 1.0000 E: NO

```

```

-----
Specimen # : 1               Test end status : 10
Maximum load : 414.305      Max load point # : 235
Max extens : .140          Max extens pnt # : 420
| 2nd. Speed | Extn. Remv | Relx Strt | Range Chg |
Point #      ---      ---      ---      ---
Number of points : 420
Specimen dimensions :
A: .2500000 B: .0559055 C: 1.0000000 D: 1.0000000 E: NO

```

Transverse gauge: -----

Auxiliary Specimen Inputs:

Auxiliary Sample Inputs:

Calculations from Instron:

```

|----- Maximum -----|----- Break -----|
|Load   Displ.   Strain |Load   Displ.   Strain |
|414.32 .84252E-01*****|*****          *****|
|----- Peak 1 -----|
|Load   Displ.   Strain |
|*****          *****|

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1, .000060, .000000
2, .000390, .000000
3, .000720, .000000
4, .001140, .000000
5, .001470, .000000
6, .001720, .000000
7, .002050, .000000
8, .002470, .000000
9, .002720, .000000

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Appendix C - Sample Spreadsheet for 7075 Aluminum

This appendix shows the summary page of the spreadsheet used in the analysis of the tensile test data for 7075-T651 aluminum. Measured values are generally close to the reference values. The accuracy of the analysis of the strain hardening behavior was improved somewhat by assuming a prior strain of 0.02.