PLASTIC DEFORMATION AND THE ONSET OF TENSILE INSTABILITY

Introduction

In this experiment the plastic deformation behavior and the onset of plastic instability of some common structural alloys is investigated. The usual mechanical properties are measured from stress-strain curves, but in addition, strain hardening behavior and the onset of plastic instability are studied. A lot is going on during a tensile test, some having important practical uses and many also hinting at the fundamental mechanisms of deformation processes. The goal of this experiment is to investigate all of them.

Background

Elastic Deformation When an isotropic material is loaded to stresses below the yield point and then unloaded again all of the deformation is immediately recovered. This behavior is usually associated with the linear portion of the stress-strain curve, although not all materials exhibit this linear-elastic behavior, rubber for instance. But for many materials the resistance to deformation in this region is given by Young's modulus E

$$E = \frac{d\sigma}{d\epsilon}.$$

When measuring Young's modulus one must take into account the stiffness of the testing machine. Ideally, the machine would be perfectly stiff but in reality it may be in the neighborhood of 40,000 to 100,000 pounds per inch. This will introduce errors in the measured modulus which are given by

$$E_{measured} = E\left[1 + \frac{A_0 E}{K L_0}\right]$$

where A_0 is the initial cross-sectional area of the specimen, L_0 is the initial gage length, E is the correct value of young's modulus and K is the stiffness of the testing machine.

Without these corrections, the error in the measured modulus is mostly in measurement of elongation. The specimen, being a monolithic part with no joints while the testing machine has many joints, moving parts that mesh better when loaded, etc., means the specimen is stiffer than the testing machine. Consider also that the modulus is the ratio of stress, a large number, to strain, a small number. The result is very sensitive to errors in the denominator.

When conducting a constant crosshead speed test one can compute the stiffness of the testing machine using the equation

$$K = \left(\frac{\dot{x}}{\dot{F}} - \frac{L_0}{A_0 E}\right)^{-1}$$ Machine Stiffness

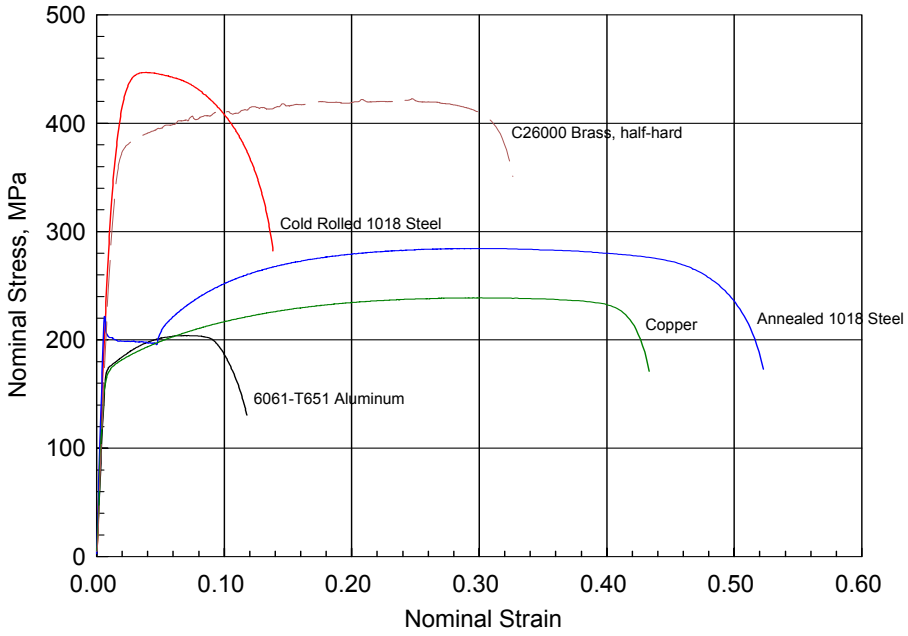

Figure 1. Stress-strain curves for several common alloys.

where $\dot{x}$ is the crosshead speed and $\dot{F}$ is the loading rate. Or, one can use an extensometer to measure strain directly from the specimen. If, after the test, one compares this strain to the displacement of the crosshead one will notice small errors in the engineering strain rate.

Yielding: Materials scientists recognize that some plastic deformation is occurring prior to the yield point and that elastic deformation continues past the yield point. In other words, yielding does not happen all at once at a single point on the stress-strain curve, as might be thought when considering yielding from the continuum mechanics point of view. A more practical definition of yielding is when deformation changes from being mostly elastic to mostly plastic. This definition seems to make the method of measuring the yield strength somewhat arbitrary, which it is, but standard methods have been defined so that everyone will get the same result. In fact, there are a number of standard methods one may use and the method used varies depending on the type of yielding behavior (sudden, gradual, upper/lower), industry, or country. In any case, the significance of the yield point is that prior to yielding most of the deformation was recoverable elastic deformation and after yielding most of the additional deformation is permanent, plastic deformation.

Strain Hardening: Plastic deformation can occur by three different processes: diffusion, twinning and dislocation motion. The contribution of diffusional processes is important at high homologous temperatures, $T > 0.6\, T_{mp}$. Twinning might account for up to a few per cent strain while dislocation processes are generally the principle mechanisms for plastic deformation.

Considering only the contribution of dislocations, the shear stress acting on the slip plane causes these dislocations to move through the lattice. As they move they may encounter obstacles such as solute atoms, particles, grain boundaries and other dislocations, effectively stopping the dislocation until the stress is increased and the dislocation can overcome the obstacle. While this dislocation was gliding, climbing and cross-slipping through the lattice additional dislocations were being

generated and they were moving through the lattice. With this ever increasing dislocation density the distance between them decreases, they encounter each other more often, and it becomes increasingly difficult to deform the material. This process is called strain hardening.

There are two well known equations that depict the strain hardening phenomenon. The Hollomon equation describes strain hardening as a power law function of stress and strain after yielding

$$\sigma = K\epsilon^n \qquad \text{Hollomon Equation}$$

where F and , are stress and strain beyond the yield point. The second equation is Ludwik's equation. It is often preferred because it includes the stress up to the yield point, not just the stress produced by strain hardening.

$$\sigma = \sigma_0 + K\epsilon^n \qquad \text{Ludwik's Equation}$$

In both of the above equations K is the strength index, n is the strain hardening index, and F_o is the yield stress. The strength index K is structure dependent and varies with processing. The strain hardening index is a constant for a given material and can range from 0 to 1 but is typically in the range of 0.2 to 0.5. In cases where the material has already experienced some plastic deformation the following equation represents strain hardening

$$\sigma = K(\epsilon + \epsilon_0)^n.$$

The strain hardening index can be written as

$$n = \frac{d\log(\sigma)}{d\log(\epsilon)} = \frac{\epsilon}{\sigma}\frac{d\sigma}{d\epsilon} \qquad \text{Strain Hardening Index}$$

and it can evaluated numerically or graphically from the slope of a plot of log(F) -vs- log(,). This equation also expresses strain hardening in terms of the rate of increase of stress with respect to strain. Rewriting it gives

$$\frac{d\sigma}{d\epsilon} = n\frac{\sigma}{\epsilon} \qquad \text{Rate of Strain Hardening}$$

which says that the rate of strain hardening is proportional to the stress divided by strain.

Strain Rate Hardening: Most materials are sensitive to the rate of deformation. This behavior often obeys a power law expression such as

$$\sigma = A\dot{\epsilon}^m \qquad \text{Dorn Equation}$$

where A is a constant at given temperature and metallurgical condition and m is the strain rate sensitivity. This equation can be combined with the Hollomon equation for a general description of the plastic deformation of materials.

$$\sigma = B\dot{\epsilon}^m\epsilon^n.$$

At low and ambient temperatures the rate of strain hardening is significant, 0.2<n<0.5, while the rate sensitivity is small, 0.001<m<0.01. At higher temperatures just the opposite is true and the strain rate sensitivity can reach values as high as 1.0 (Newtonian viscosity) while the strain hardening index can approach zero.

While the strain rate sensitivity can effect the flow stress it is more important in terms of its effect on ductility. While increased strain rate sensitivity slightly increases the strain at which the material starts to neck, it can extend ductility considerably by stabilizing the neck. As the specimen begins to neck the strain rate in the necked region increases. The strain rate sensitivity causes the stress in this region to increase and thus the neck stabilizes and ductility is extended. So while the strain hardening index was a measure of a material's resistance to the onset of instability, the strain rate sensitivity is a measure of its resistance to necking.

Tensile Strength: The tensile strength F_u is important from both design and scientific viewpoints. It is related to the maximum nominal stress (UTS) that a material can sustain in tension, it occurs at the point where necking begins, and it has a number of interesting and useful empirical relationships to other properties. If a true stress-true strain curve could be obtained directly from a tensile test, instead of an engineering stress-strain curve, one would find that there is no maximum stress prior to failure and therefore there would appear to be no UTS. That is not to say that necking has not occurred or that there is no UTS, but that other methods must be employed to determine F_u.

There are several methods for determining the tensile strength from a true stress-strain curve. One method is to plot both the true stress and the rate of strain hardening against true strain as shown in figure 2. The point where they intersect marks the tensile strength. This method, derived from the Hollomon equation, is expressed in the following equation

$$\sigma = \sigma_u \text{ when } \frac{d\sigma}{d\epsilon} = \sigma . \qquad \text{Tensile Strength}$$

A second method of determining the tensile strength is the unit subtangent method. Using this

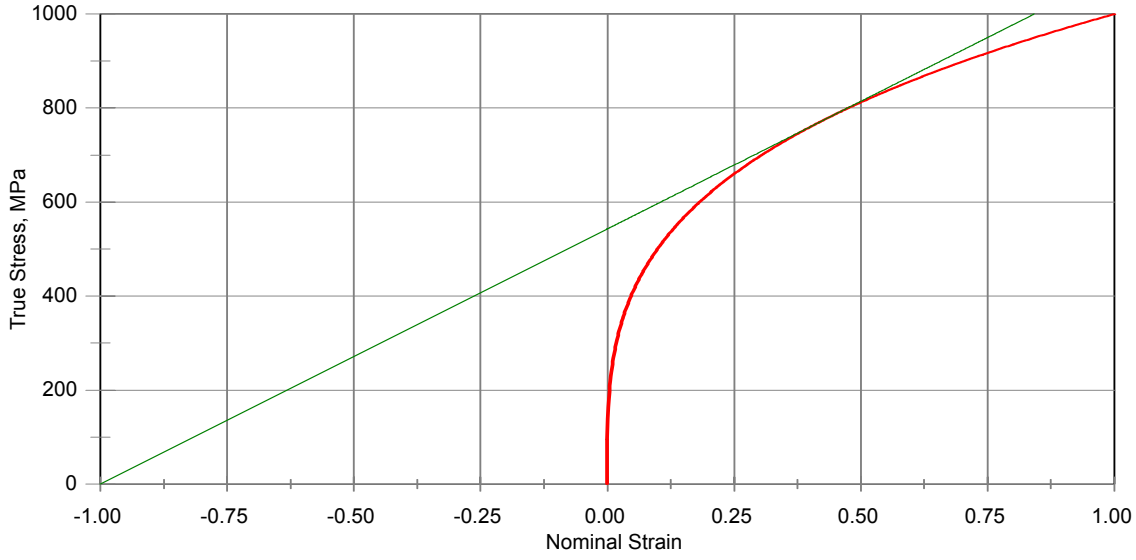

Figure 2. Considère's construction for determining the tensile strength of a ductile material. The tensile strength is found at the intersection of the straight line and the stress axis at strain=0. [2]

method the tensile strength can be found by locating a point on the true stress-true strain curve which has a subtangent of unity. Furthermore, it can be shown that

$$\sigma = \sigma_u \text{ when } \frac{d\sigma}{de} = \frac{\sigma}{1+e} \qquad \text{Considere's Criterion}$$

which is Considere's criterion, giving us a third method for determining the tensile strength. It is also a graphical method but it requires a plot of true stress versus nominal strain whose origin is set at the yield point. The tensile strength is obtained by drawing a subtangent to a nominal strain of -1. The UTS is the point where this line intersects the stress axis. (See figure 3.)

If power law hardening is observed then Considere's criterion can also be used to derive one last expression for the tensile strength

$$\sigma_u = Kn^n(1-n)^{1-n}. \qquad \text{Tensile Strength}$$

Onset of Plastic Instability: Generally, the strain at maximum load marks the onset of plastic instability. For annealed materials the rate of strain hardening immediately after yielding is relatively high but as deformation continues the rate of strain hardening steadily decreases. At the same time the specimen is becoming thinner. Eventually the rate of strain hardening and specimen thinning are equal and this is the point where deformation becomes unstable. (This is also the point where the increasing load carrying capability due to strain hardening and the decreasing load carrying capability due to geometric softening produce a maxima on the load-elongation curve.) Beyond this point the rate of specimen thinning is greater than the rate of strain hardening and deformation is unstable. The instability will start in some part of the specimen where an

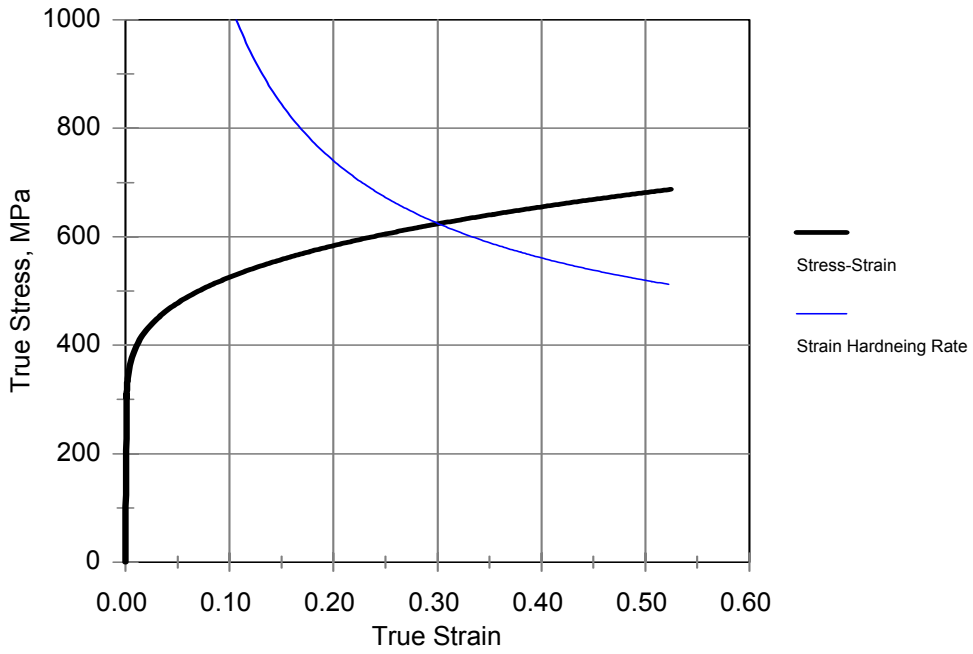

Figure 3. Stress-strain and rate of strain hardening curves calculated using the Hollomon equation (K=500 MPa, n=0.3)

inhomogeneity (soft spot, adjacent to a hard spot) or at a machining defect (stress concentration) exists. Once localized deformation begins the stress in this region will be higher than elsewhere, causing deformation to become even more localized, leading to the formation of a macroscopic neck.

For rate insensitive materials, the onset of unstable deformation occurs at the strain where maximum load is obtained. It was mentioned earlier that the strain hardening index is a measure of the resistance to the onset of instability. It can be shown that the strain hardening index for rate insensitive materials is equal to the true strain where necking begins.

$$\epsilon_u = n \qquad \text{Onset of Necking}$$

For rate sensitive materials the onset of necking will differ but only slightly. The most important aspect of rate sensitivity is that it tends to stabilize the neck, decreasing its degree of localization, and extending ductility well past $,_u$.

Ductility and Localized Deformation: During nonuniform deformation stresses are concentrated in the necked region of the specimen, leading to localized deformation and to a more complex stress state. Bridgeman [1] has developed a method for computing the stress distribution in the necked region and making corrections to the stress-strain data based on the radius and area at the root of the neck, figure 4.

The onset of instability often marks the limit of useful deformation. However, neck growth is slow at first and additional straining can occur before the instability is either noticeable or significant. Useful ductility beyond maximum load is affected primarily by the strain rate sensitivity of the material. For example, maximum load is often reached at about 10-15% strain, which is usually around half the strain to fracture for rate insensitive materials (m - 0.01). However, superplastic materials have high rate sensitivities, *m* having a value close to 0.5, and often deform in a stable manner to strains greater than 400% and sometimes over 2000%. The maximum load, however, occurred at around 10% strain. The high rate sensitivity stabilized the neck for up to 20 times that amount of strain and the necks that did form are usually diffuse.

Fracture: With continued plastic deformation the material accumulates damage associated with dislocation processes. Dislocations that pile up at grain boundaries and other obstacles can form microcracks. Tensile stresses can open these into voids which can grow and eventually coalesce, forming larger cavities. Final fracture occurs when the number and sizes of internal voids is sufficient to exceed the material's ability to withstand the applied load. These voids introduce stress concentrations which can lead to the spontaneous growth of a single crack. (This behavior is the subject of the field of fracture mechanics.) Inspection of the fracture surfaces permits determination of the type of fracture process involved, brittle, shear, or ductile. Close inspection of the cup and cone fracture surfaces of a ductile material may reveal areas where void formation and coalescence occurred and the area through which the final rupture occurred. A scanning electron microscope is usually required to determine whether the crack propagated intergranularly or transgranularly and how much plastic deformation occurred as the crack propagated.

The strain to fracture is not a property of the material. It also depends on the specimen geometry. An expression that relates specimen geometry to ductility is

$$\epsilon_f = C\frac{A_0^{1/2}}{L_0}$$

where A_0 is the original cross-sectional area and L_0 is the original gage length. Ideally, comparisons of strain to fracture should be made using specimens of identical design. If this isn't possible then the comparisons should be made on similarly shaped specimens. Corrections based on the $A^{1/2}/L$ ratio can be made. ASTM 370 describes how to make comparisons of round and flat specimens.

Preparation

1. Plot true stress true strain for a fictitious material using either the Hollomon or Ludwik equation. Experiment with the values of K and n to see how these effect the results. Note the stress as true strain approaches 100%.

2. Determine the tensile strength for the stress-strain curve in the question above.

3. Compile mechanical properties data on the materials to be tested.

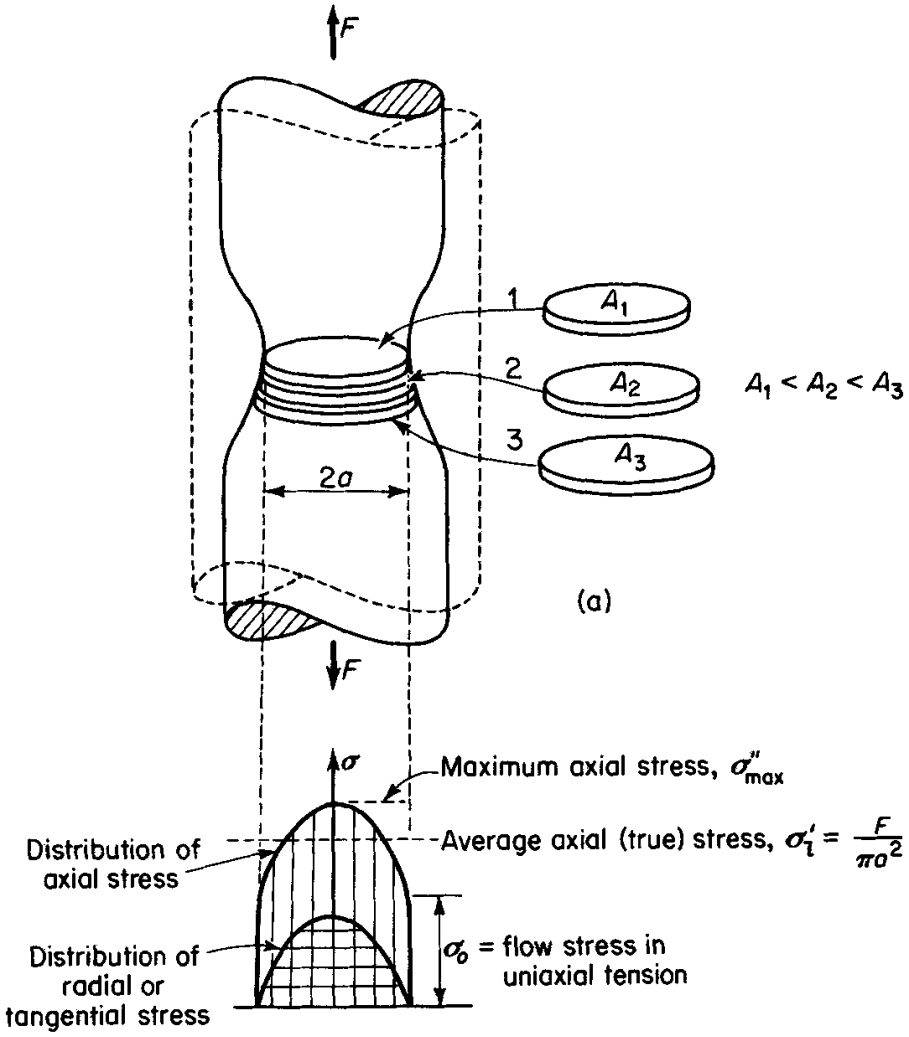

Figure 4. Distribution of stresses in the necked region of a tensile specimen [3].

4. Note the specifications of the tensile tester and compare them to the load and elongation requirements for the tensile tests.

5. For the crosshead speed you plan to use, determine the data acquisition rate you will need to be able to record enough data in the elastic region to be able to determine Young's modulus.

Safety Considerations

During this experiment high forces are generated by the tensile testing machine. It has pinch zones large enough to trap a finger, a hand and even an arm. Injuries, if they should occur, could be quite serious. A detailed operating procedure is included in this laboratory manual. Read it before you start using this equipment and refer to it during every step of the experiment. Be especially careful when installing a specimen, stay clear while the test is running and keep in mind that some types of specimens may shatter when broken.

Chemical Hazards	Normally none, but this will depend on the materials the specimens are made from. Specimens used in this experiment are usually made of steel, copper, brass, aluminum alloys or other conventional structural materials.
Physical Hazards	Tensile testing machines can generate tens of thousands of pounds force. Be very careful when installing a specimen and stay back when the test is running. Brittle specimens and composites tend to send small debris flying about the room when the specimen breaks. Compression testing any material or structure poses the same hazard. If these types of specimens are used or if you plan to do a compression test then either a scatter shields should be installed on the load frame or everyone should be wearing safety glasses.
Bio-hazards	None.
Radiation Hazards	None.
Protective Equipment	Recommended: safety glasses Required: safety glasses and/or scatter shields if compression testing is done or if brittle materials or composites are tensile tested.

Materials

Specimens made of plain carbon and low alloy steels, aluminum, and plastics are available. All specimens are round with threaded ends. They all have a 2 inch gage length and either an 0.236 or 0.505 inch diameters.

Procedure

Inspect the equipment and become familiar with how it operates. Check the calibration of the load cell and extensometer and make adjustments as required.

Select a specimen to test and measure and record its gage length, diameter, etc. Note the specimen's

composition and processing and estimate the maximum load that will be generated during the tensile test.

Tensile test the specimen to failure. During the test monitor the load and elongation and watch the specimen carefully to see when necking begins.

After the specimen fractures, remove the broken halves and note the geometry of the fracture and the fracture surfaces. Do not try to fit the halves of the specimen back together again. This will damage the fracture surfaces. You'll probably want to examine the fracture surfaces under a low power microscope or a scanning electron microscope.

Finally, measure final length and diameter of the specimen.

Analysis

Perform a comprehensive analysis of the data from the tensile tests. Measure all of the usual mechanical properties plus the energy capacity of the material, and then analyze the strain hardening behavior. Finally, investigate the onset of tensile instability.

Stiffness: Determine the value of Young's modulus from both the load-elongation data. How does it compare to the published values?

Repeat this analysis again but this time use the load-time (constant crosshead speed) data. Compare this value to the previous value. This value is typically a factor of 10 to 20 lower than the value obtained using an extensometer. Determine the stiffness of the testing machine.

General Mechanical Properties: Compute the other basic mechanical properties that can be obtained from a tensile test.

- Stress
  - Yield strength, (upper and lower, offset, etc.)
  - Tensile strength (stress at maximum load)
  - Stress at fracture
- Strain
  - Strain to cause yielding
  - Strain to cause necking
  - Strain to cause failure (ductility)
  - Reduction of area at failure
- Energy capacity
  - Work required to cause failure
  - Modulus of resilience
  - Modulus of toughness

Compare these properties to those in reference books.

Strain Hardening: Use a least squares or regression analysis to measure the parameters of the Hollomon or Ludwik equation. Comment on the values obtained. Are these typical values? Are either of these equations satisfactory for describing the stress-strain behavior of these materials?

Tensile Strength: Compute the tensile strength using the load-elongation or engineering stress-strain data. Next, determine the tensile strength using the rate of hardening method. Finally, locate the tensile strength on the true stress-strain curve using only the value of the strain hardening exponent. Do both methods give the same answer? Are either values that same as the true stress at the UTS that was obtained from the nominal stress-strain curve?

Onset of Instability: When did you first observe necking in the specimen? Compare this to the strain where maximum load (tensile strength) occurs on the stress-strain curve. Do the Hollomon strain hardening index and strain at the onset of instability coincide?

Fracture: Examine the fracture surfaces of each specimen. Did fracture occur in a ductile or brittle manner?

Measure the final cross-sectional area of the fractured specimen and compute the true stress at fracture. Compare this to the engineering stress that was obtained at the same strain and the true stress that the Hollomon equation would predict. One would expect it to be lower. Why? Was it?

References

1. Bridgeman, P. W., Transactions of ASM, v. 32, pp. 553-574, (1944).

2. Wyatt, Oliver H. and Dew-Hughes, David, Metals, Ceramics and Polymers, Cambridge University Press, London, p. 143, (1974).

3. Polakowski, N.H. and Ripling, E.J., Strength and Structure of Engineering Materials, Prentice-Hall, New Jersey, (1966).

Further Reading

1. Mechanical Testing, Metals Handbook, Volume 8, ASM International, Metals Park, OH, (1985).

2. Smithells Metals Reference Book, sixth edition, ed. Eric A Brandes, Butterworths & Company, London, (1983).