## The Slope of a Line on a Graph

## Introduction

Imagine you or your colleague have performed a series of difficult and important but also careful and accurate measurements and then plotted them so that you can see the trend. In your analysis of the trend you draw a straight line through it so that you can measure the slope of the line. In this process you will probably find yourself making personal judgements, based on your experience in these matters, as to the validity and accuracy of the data, whether or not a particular data point belongs in the trend, etc. How do you make these judgements without biasing the results? How do you maintain your objectivity when you "know" what the final result could or should be? Are there better ways to do this?

Occasionally, while listening to a technical presentation or reading a scientific paper one comes upon a graph showing data points with a line drawn through them that indicates how the presenter interprets the data, and you, and possibly others in the audience, insist you would have drawn the line differently and therefore would have come to a different conclusion. Welcome to the real scientific world.

Let's consider a case involving the kinetic of grain growth in an alloy. The rate of grain growth is often described by the equation

$$
\begin{equation*}
\tau^{n}-d_{o}^{n}=k t \tag{1}
\end{equation*}
$$

where $d$ is the grain size at time $t, d_{0}$ is the initial grain size, $k$ is a function of temperature and other factors, and $n$ is a constant which, for the case of ideal grain growth, is equal to 2 . We can simplify this by assuming that $d_{0}$ is equal to 0 to get

$$
\begin{equation*}
d^{n}=k t \tag{2}
\end{equation*}
$$

and if we take the $\log$ of both sides of the equation we can rewrite it as

$$
\begin{equation*}
n \log (d)=\log (k)+\log (t) \tag{3}
\end{equation*}
$$

and finally if we then divide through by $n$ to obtain the following

$$
\begin{equation*}
\log (d)=\frac{1}{n} \log (k)+\frac{1}{n} \log (t) . \tag{4}
\end{equation*}
$$

This equation is in the form of an equation for a straight line, $y=b+m \mathrm{x}$, for which the slope $m$ of a $\mathrm{y}=\log (d)$ versus $\mathrm{x}=\log (t)$ plot would be equal to $1 / n$ and the intercept b would be $1 / n \log (k)$. The
goal in this study was to determine the value of $n$ to see if the data shows that this is a case ideal grain growth.

## Procedure

Examine the data in the accompanying plot. Draw what you think is the best straight line that represents the trend in the data. Determine the slope of this line and calculate the value of $n$.

## Results and Analysis

1. Did you feel obligated to include all of the data in the trend?
2. Did you discard or ignore any of the data when you drew the line?
3. When is it acceptable to "throw out" data?
4. How comfortable did you feel evaluating this data when you knew so very little about how the data was generated? Should it even matter?
5. How objective do you think your analysis was? How could you make the analysis more objective?
6. Does this data show that this is a case of ideal grain growth?
7. Does everyone else agree with you? If not, how would you proceed?

## Conclusions

What do you conclude from all of this?


Figure 1. Grain size as a function of annealing time for $70 / 30$ brass. Annealing was done at $650^{\circ} \mathrm{C}$.

